Exercise 27

Calculate y'.

$$y = \log_5(1 + 2x)$$

Solution

Method 1 - Using a Differentiation Formula

Recall that the derivative of a logarithm to base a is

$$\frac{d}{dx} \log_a x = \frac{1}{x \ln a}.$$

Calculate y' by using the chain rule.

$$\begin{aligned} y' &= \frac{d}{dx} \log_5(1 + 2x) \\ &= \frac{1}{(1 + 2x) \ln 5} \cdot \frac{d}{dx}(1 + 2x) \\ &= \frac{1}{(1 + 2x) \ln 5} \cdot (2) \\ &= \frac{2}{(1 + 2x) \ln 5} \end{aligned}$$

Method 2 - Changing to a Natural Base

Knowing the derivative of a logarithm with an unnatural base isn't necessary.

$$y = \log_5(1 + 2x)$$

Exponentiate both sides with base 5.

$$5^y = 5^{\log_5(1+2x)}$$

$$5^y = 1 + 2x$$

Solve for y by taking the natural logarithm of both sides.

$$\ln 5^y = \ln(1 + 2x)$$

$$y(\ln 5) = \ln(1 + 2x)$$

$$y = \frac{\ln(1 + 2x)}{\ln 5}$$

Now take the derivative of both sides with respect to x.

$$\frac{d}{dx}(y) = \frac{d}{dx}\left[\frac{\ln(1+2x)}{\ln 5}\right]$$

$$\frac{dy}{dx} = \frac{1}{\ln 5}\frac{d}{dx}\ln(1+2x)$$

$$= \frac{1}{\ln 5}\left(\frac{1}{1+2x}\right)\cdot\frac{d}{dx}(1+2x)$$

$$= \frac{1}{\ln 5}\left(\frac{1}{1+2x}\right)\cdot(2)$$

$$= \frac{2}{(\ln 5)(1+2x)}$$